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TESTING OF THE $\overline{v^2}$–f MODEL OF TURBULENCE IN CALCULATING THE FLOW AND HEAT TRANSFER IN AN ABRUPTLY EXPANDING DUCT

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This paper presents the results of testing of the $\overline{v^2}$–f model of turbulence with the examples of two-dimensional flow of an incompressible fluid and heat exchange with the walls of an abruptly expanding duct. The calculation data have been compared to the known experimental data, and the data of direct numerical simulation, as well as to results of calculations with the use of two versions of the k–ω model.

1. Modern engineering hydrodynamic calculations, including convective heat-transfer problems, are based mainly on the solution of Reynolds-averaged Navier–Stokes equations jointly with the chosen turbulence model determining the eddy viscosity. However, despite the large number of known models of turbulent transfer, it is impossible to single out one of them that outperforms the others for reliability and universality as applied to the wide class of flows. The semiempirical models of eddy viscosity that have successfully stood the test on steady flows in ducts or low-gradient boundary layers can work unsatisfactorily in the case of practical problems with a complex geometry, separated flows, recirculation zones, etc. As a consequence, for each particular turbulence model it is of interest to know the types of flows that it describes with a reasonable accuracy.

Recently, Durbin's $\overline{v^2}$–f model [1] has become popular, as it has shown good results in engineering calculations of a number of internal flows with heat transfer. In particular, it provides a good rendition of the heat-transfer coefficient over a turbine blade surface [2] and on the wall of a periodically ribbed duct [3]. The $\overline{v^2}$–f model was also put to an evaluation test on flows in abruptly expanding ducts, which was a canonical test for estimating the acceptibility of semiempirical turbulence models for calculating the hydrodynamics and heat transfer in flows with separation and reattachment. It has been established that the model provides a fair accuracy of rendition of the hydrodynamic characteristics of such flows [1, 4]. However, no publications containing results on applying the $\overline{v^2}$–f-model to the calculation of the heat transfer in abruptly expanding ducts are known to the author.

The present paper explores the possibility of predicting the basic characteristics of flow and heat transfer in an abruptly expanding duct. The results of the calculations have been compared to the experimental data of [5], as well as to the data obtained with the use of two versions of the k–ω model.

In numerical investigations, we used the SINF program complex developed at the hydrodynamics subfaculty of the St. Petersburg Polytechnic University, which makes it possible to calculate multidimensional stationary and nonstationary flows in complex-geometry regions. For turbulent transfer simulation, the complex provides a large number of high- and low-Reynolds models of effective viscosity, including the families of k–ε and k–ω models, Spalart–Allmaras and Wolfstein models, as well as the $\overline{v^2}$–f model. The correctness of the program realization of the latter, done by us, was checked, in particular, in solving the problem on development of a boundary layer on a flat plane, as well as the problem on flow in an abrupt-expansion duct at a low Reynolds number.

2. To describe the nonisothermal flow of an incompressible fluid, the system of stationary equations of continuity, motion, and energy is used:

$$\nabla\cdot\mathbf{V} = 0 \,, \tag{1}$$

$$(\mathbf{V}\cdot\nabla)\,\mathbf{V} = -\,\nabla p + \nabla\cdot\,(2\nu_{\text{eff}}\dot{S}) \,, \tag{2}$$

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$$(\mathbf{V}\cdot\nabla)\, T = \nabla\cdot\, (a_{\text{eff}}\nabla T)\,, \tag{3}$$

where $\dot{S} = \dot{S}_{ij}$ is the strain-rate tensor; $\nu_{\text{eff}} = \nu + \nu_t$ is the effective viscosity; $a_{\text{eff}} = \nu/\text{Pr} + \nu_t/\text{Pr}_t$; Pr and Pr_t are, respectively, the molecular and turbulent Prandtl numbers. In the present paper, the Pr_t value was assumed to be equal to 0.9.

According to [3], the stationary equations of the $\overline{v^2}$–f model of turbulence are of the form

$$(\mathbf{V}\cdot\nabla)\, k = P_k - \varepsilon + \nabla\cdot((\nu + \nu_t)\, \nabla k)\,, \tag{4}$$

$$(\mathbf{V}\cdot\nabla)\, \varepsilon = \frac{C_{\varepsilon 1} P_k - C_{\varepsilon 2}\varepsilon}{t} + \nabla\cdot\left(\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\nabla\varepsilon\right), \tag{5}$$

$$(\mathbf{V}\cdot\nabla)\, \overline{v^2} = kf - N\,\overline{v^2}\,\frac{k}{\varepsilon} + \nabla\cdot\left((\nu + \nu_t)\, \nabla\overline{v^2}\right), \tag{6}$$

$$f - l^2\Delta f = (C_1 - 1)\,\frac{2/3 - \overline{v^2}/k}{t} + C_2\,\frac{P_k}{k} + (N-1)\,\frac{\overline{v^2}}{kt}\,. \tag{7}$$

Here $P_k = 2\,\nu_t\dot{S}_{ij}\,\dot{S}_{ij}$ is the kinetic energy generation; the turbulent viscosity is defined as $\nu_t = C_\mu\overline{v^2}t$; t and l are the turbulent time and length scales calculated by the formulas

$$t = \max\left(\frac{k}{\varepsilon},\; 6\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right), \tag{8}$$

$$t = C_l \max\left(\frac{k^{3/2}}{\varepsilon},\; C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right). \tag{9}$$

The boundary conditions for solid walls ($Y = 0$) are as follows:

$$k\,(0) = 0\,,\quad \overline{v^2}\,(0) = 0\,,\quad \varepsilon \xrightarrow[Y\to 0]{} \frac{2\nu k}{Y^2}\,,\quad f \xrightarrow[Y\to 0]{} \frac{4\,(6-N)\,\nu^2\,\overline{v^2}}{\varepsilon Y^4}\,. \tag{10}$$

Experience in using the $\overline{v^2}$–f model has shown that its original version [1] with $N = 1$ leads to computational difficulties that arise because of the strong nonlinear influence on one another of the sought variables through the boundary condition for f. Later, the so-called "code-friendly" modification of the model of [6] with $N = 6$ and a zero boundary condition for f on a solid wall, as follows from (10), was proposed.

It should particularly be noted that by now in articles devoted to formulations of the $\overline{v^2}$–f model and its applications different versions of the set of empirical constants of the model have appeared; however, in recent papers, the following set determined for the version of the model with $N = 6$ has been used most frequently:

$$\begin{gathered} C_\mu = 0.22\,,\quad C_{\varepsilon 1} = \left(1 + 0.045\,\sqrt{k/\overline{v^2}}\right),\quad C_{\varepsilon 2} = 1.9\,,\quad \sigma_\varepsilon = 1.3\,, \\ C_1 = 1.4\,,\quad C_2 = 0.3\,,\quad C_l = 0.23\,,\quad C_\eta = 70\,. \end{gathered} \tag{11}$$

The results presented in this paper have been obtained on the basis of the modified version of the model (N = 6) with the set of constants (11).

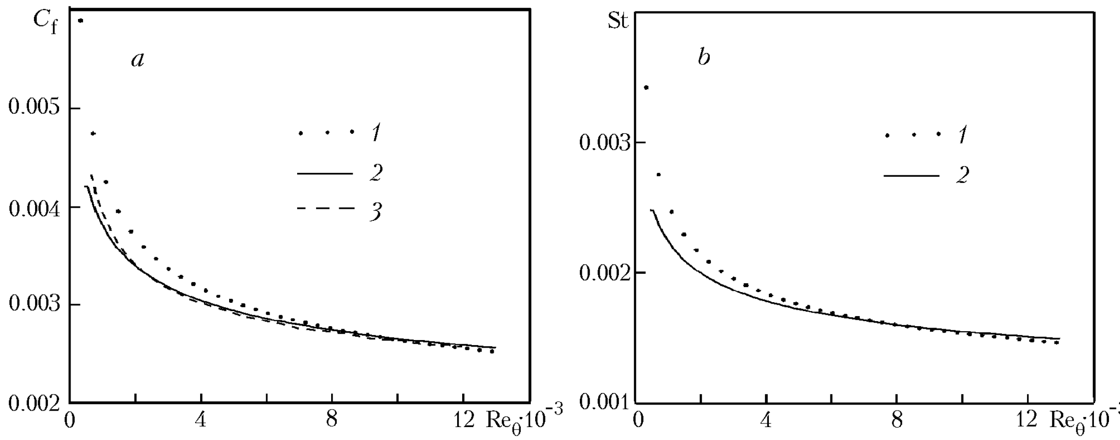

Fig. 1. Results of calculation of the turbulent boundary layer on a flat plate: 1) by empirical formulas; 2) data of the present work; 3) data of [1]; a) friction-coefficient distribution; b) Stanton number distribution.

3. In the SINF program complex, the numerical method is based on the use of multiblock structured grids matched with the flow-region boundaries. The equations of motion are written in terms of the Cartesian velocity components. The iteration process of solving the equations is based on the artificial compressibility method. In each iteration, exchange of values of calculated variables along the surfaces on which calculation grid blocks are joined takes place. In so doing, the concept of an auxiliary virtual block is used, which provides complete transparence of interblock boundaries and retention of conservative properties of the difference scheme. Discretization of the spatial operators of the conservation equations has been carried out by the finite-volume method with second order of accuracy. The values of the sought quantities are determined at the centers of control volumes. Of the options offered by the program complex for calculating convective terms, we chose the QUICK upstream scheme of [7].

4. Before performing calculations of turbulent separation flows in abruptly expanding ducts, we considered the relatively simple problem on the development of a nonisothermal boundary layer on a flat plate. Its solution was aimed, first of all, at checking the validity of our realization of the $\overline{v^2}$–f model in the SINF program complex.

To solve the problem, we used a rectangular calculation region of length L and height $0.23L$ covered with a grid with 121×97 cells. The grid nodes were crowded together toward the plate so that the Y^+ values for the centers adjoining the cell walls did not exceed unity.

The calculation was performed for $\mathrm{Re}_L = 10^7$. The heat exchange with the plate was simulated with a Prandtl number Pr = 0.72, which corresponds to the air flow. At the input boundary of the calculation region, the conditions of homogeneous velocity U_{in} and temperature T_{in} were imposed. The input values of the turbulence characteristics were as follows: the kinetic energy of turbulence k_{in} was given on the assumption of a low level of external turbulence (~1%), $\overline{v^2_{\mathrm{in}}} = 2k_{\mathrm{in}}/3$, $f_{\mathrm{in}} = 0$, the value of $\varepsilon_{\mathrm{in}}$ was chosen so that the input turbulent viscosity exceeded the laminar viscosity by a factor of 4.5. Under such conditions, the influence of external turbulence on the development of a turbulence boundary layer is immaterial; it only affects the position of the laminar–sturbulent transition. The temperature of the wall (plate) T_{w} was assumed to be constant.

Figure 1 shows the distribution over the plate of the friction coefficient C_{f} and the Stanton number St. Here U_{in} and T_{in} were taken as U_0 and T_0, respectively. The obtained calculated dependences of C_{f} and St on the Reynolds number determined by the momentum thickness have been compared to the experimental relations that hold for turbulent boundary layers with a low degree of external turbulence. To calculate C_{f}, we used the widely known Bradshaw empirical formula [8]

$$C_{\mathrm{f}} = \frac{0.01013}{\log \mathrm{Re}_\theta - 1.02} - 0.00075\,, \tag{12}$$

and the formula for the Stanton number calculation

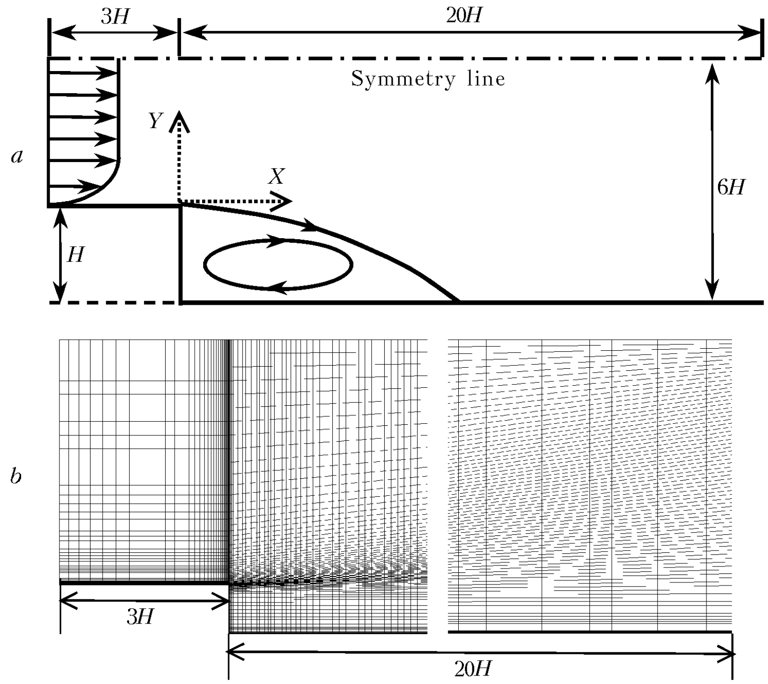

Fig. 2. Calculation region (a) and grid (b) used in simulating the isothermal flow in a symmetric abrupt-expansion duct.

$$\mathrm{St} = 0.5KC_\mathrm{f} \tag{13}$$

was obtained from (2) at a Reynolds empirical analogy number K = 1.16 [8]. Figure 1 also shows the friction coefficient distribution obtained by Durbin in [1] in solving an analogous problem but with the use of the original version of the $\overline{v^2}$–f model. It is seen that for the boundary layer on the plate both versions of the $\overline{v^2}$–f model yield underestimated values of the friction coefficient and the Stanton number at small Reynolds numbers Re_θ, and at $\mathrm{Re}_\theta < 10{,}000$ their slight overestimation is observed. The calculated distributions for different versions are in good agreement with one another, and only in the region of $\mathrm{Re}_\theta < 1500$ does the original version yield a somewhat higher friction. It should be remembered here that the model modification was aimed, first of all, at elimination of the computational difficulties rather than at improvement of the accuracy of calculation of canonical flows. In [6], it was shown that as applied to the calculation of the turbulent boundary layer, the "code-friendly" modification affects only the position of the laminar–turbulent transition. The results of applying the $\overline{v^2}$–f model to the problem on the development of a turbulent boundary layer presented in this paper point to its correct realization in the SINF program complex.

5. The isothermal flow in a symmetric abrupt expansion duct was calculated under the conditions used in [9] for direct numerical simulation of this flow. The results of the above work are considered in the literature as standard. The flow scheme, the calculation region, and the grid on which the calculations were performed in the present paper are given in Fig. 2. The grid consists of two joined blocks, the first of which contains 30 × 49 cells and the second of which has 120 × 87 cells. The Reynolds number of the flow Re_H determined by the maximal velocity at the input into the duct and the rib height is 5100. The profiles of the longitudinal velocity and turbulence characteristics at the inlet boundary were obtained from the preliminary calculation of the turbulent boundary layer developing on the wall of the intake conduit. The boundary-layer thickness was chosen so that the longitudinal velocity profile coincided with the data of the direct numerical simulation in the section $X/H = -3$.

Figure 3 gives the friction-coefficient distributions on the duct wall behind the rib (here the maximum velocity at the duct inlet is used as U_0). It is seen that the values of the friction coefficient calculated with the use of the $\overline{v^2}$–f model are basically in good agreement with the data of the direct numerical simulation, and at $X/H > 10$ their complete coincidence is observed. In the separation zone the modulus of the friction coefficient is underestimated. For comparison, Fig. 3 also shows the friction-coefficient distribution obtained in [4] in calculating the flow by means of

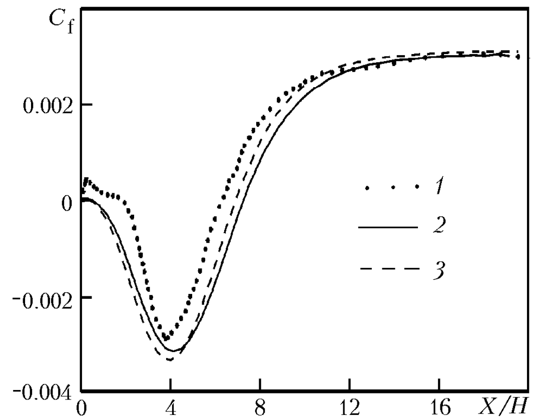

Fig. 3. Longitudinal distribution of the friction coefficient along the wall downstream from the rib of a symmetric abruptly expanding duct: 1) data of the direct numerical simulation of [9]; 2) data of the present work; 3) data of [4].

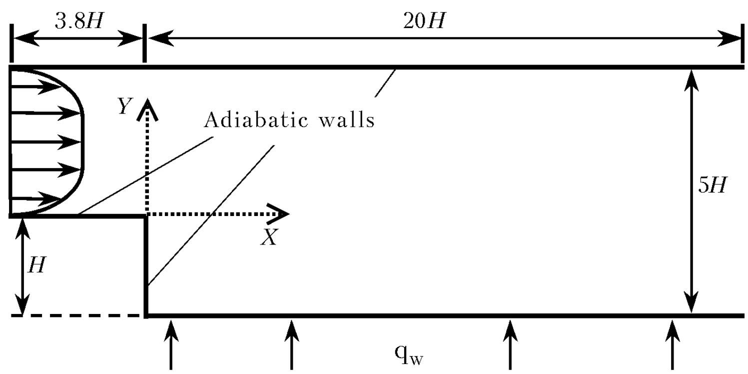

Fig. 4. Calculation region and boundary conditions used in simulating a flow with heat transfer in an abruptly expanding duct.

the commercial FLUENT program according to the modified (N = 6) version of the $\overline{v^2}$–f model. It is seen that the results obtained in the present paper and in [4] are fairly similar. Some differences observed mainly in the recirculation zone can, in particular, be due to the difference in the input velocity profiles and in the distributions of the turbulent characteristics (in [4], there is no information on the intake conduit length and the boundary conditions used in the calculations). The given parametric calculations (whose results are not presented here) have shown that the friction-coefficient distribution on the wall behind the rib largely depends on the input conditions. In general, the results of the isothermal flow calculation and the comparisons made confirm the correctness of the realization of the $\overline{v^2}$–f model in the SINF program complex.

6. The numerical simulation of the nonisothermal flow in a duct with abrupt, one-side expansion was performed in accordance with the experimental conditions of [5]. The calculation region and the boundary conditions are given in Fig. 4. The relative height of the rib is 1/4, and the Reynolds number Re_H determined by the maximum velocity at the input into the calculation region U_0 and the rib height H is 28,000. In the inlet cross section (X/H = −3.8), the distributions of the velocity and turbulence characteristics obtained as a result of the preliminary calculation of the turbulent boundary layers developing on the intake conduit walls were given. In calculating the boundary layers, it was taken into account that the value of the Reynolds number constructed by the displacement thickness in the cross section X/H = −3.8 was equal to 3370 in the experiment. Downstream from the rib, on the duct wall a constant thermal flow was maintained, and all the other walls were adiabatic. The value of the molecular Prandtl value was taken to be equal to 0.71.

For calculations, we used a grid consisting of four joined blocks with a total number of cells of 27,000. The mean value of Y^+ for the centers of wall cells was 0.2. The grid was supplied by Strelets et al. [10], who had tested on this problem a number of other turbulence models, of which Menter's model [11] yielded the best results.

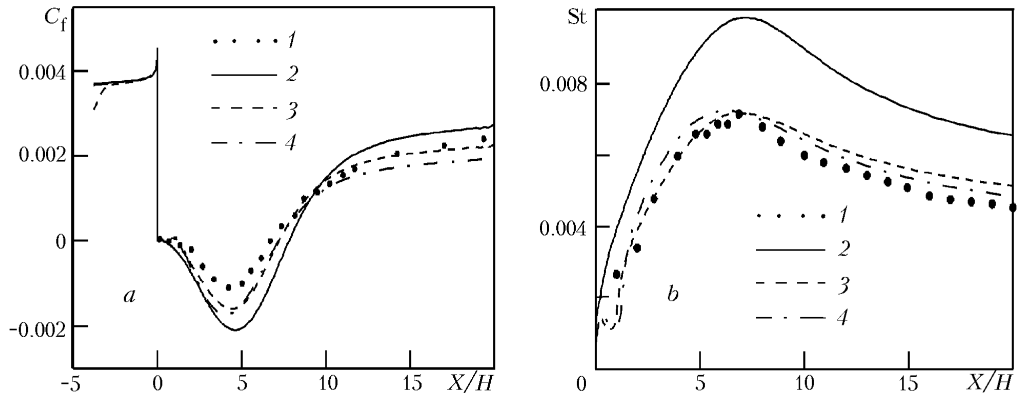

Fig. 5. Longitudinal distributions of the friction coefficient (a) and the Stanton number (b) along the wall with a rib: 1) experimental data of [5]; 2) $\overline{v^2}$–f model (data of the present paper); 3) Wilcox's k–ω model; 4) Menter's k–ω model.

Figure 5a compares the calculated and experimental data on the friction coefficient on the duct wall and behind the rib. All calculations have been performed on one and the same grid with the use of three turbulence models: the $\overline{v^2}$–f model (the calculation was done by the author of the present paper), Menter's model, and the low-Reynolds version of the k–ω Wilcox model [12] (these results were supplied to the author by A. G. Abramov, who also used the SINF program complex for his calculations). It is seen that the best agreement with the experiment is observed for the distribution obtained by Wilcox's model: the $\overline{v^2}$–f model gives two large negative values of friction in the separation region, and the reattachment point is situated far downstream, and farther downstream the calculated friction coefficient coincides with the experimental one. Noteworthily, a qualitatively similar difference from the experimental distribution of the friction coefficient also took place for the above-considered duct with symmetric expansion.

The possibility of predicting by the same turbulence models the characteristics of the heat exchange with the wall is illustrated in Fig. 5b (for the calculation of the Stanton number, the inlet temperature was used as T_0). Here Wilcox's and Menter's models demonstrate good agreement with the experiment with a slight advantage of the latter, whereas the $\overline{v^2}$–f model yields a markedly overestimated value of the Stanton number along the length of the duct wall.

CONCLUSIONS

1. We have investigated the ability of the "code-friendly" version of the $\overline{v^2}$–f model to forecast the flow and heat-transfer characteristics in the separation flow with reattachment that arises in a flat duct with abrupt expansion.

2. It has been shown that for a relatively low Reynolds number (Re_H = 5100) the model reproduces fairly well the standard distribution of the friction coefficient on the duct wall. However, at a higher Reynolds number (Re_H = 28,000) an appreciable difference from the experimental data is observed for both the friction coefficient and the characteristics of the heat exchange with the wall behind the rib.

NOTATION

a, thermal diffusivity; $C_f = 2\tau_w/(\rho U_0^2)$, friction coefficient; c_p, specific heat capacity; f, auxiliary variable of Durbin's model; H, rib height; k, kinetic energy; L, plate length; N, constant of Durbin's model defining a particular version of the model; p, pressure; $\mathrm{Pr} = \nu/a$, Prandtl number; q_w, heat flow; $\mathrm{Re}_H = U_0H/\nu$, Reynolds number for the duct; $\mathrm{Re}_L = U_0L/\nu$, Reynolds number for the plate; $\mathrm{Re}_\theta = U_0\theta/\nu$, Reynolds number for the plate constructed by the momentum thickness; $\mathrm{St} = q_w/(\rho U_0 c_p(T_w - T_0))$, Stanton number; T, normalized temperature; U, X-component of velocity; $\overline{v^2}$, variable introducing the second turbulent scale in Durbin's model; $\mathbf{V}$, velocity vector; X and Y, longitudinal and transverse coordinates; $Y^+ = (\tau_w/\rho)^{1/2}Y/\nu$, normalized distance to the wall; ε, dissipation rate of kinetic energy; ν and ν_t, kinematic and turbulent viscosity; θ, momentum thickness; ρ, density; τ, friction stress; ω, dissipative variable in Wilcox's and Menter's turbulence models. Subscripts: eff, effective; f, friction; in, inlet section; t, turbulent; w, wall; 0, characteristic value of a quantity.

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